

University of South Florida 2014

Welcome to Rocketry

An Introduction to Rocket Design Mechanics

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Abstract:

The 2014-2015 HYBRID MOTOR HIGH POWERED ROCKET COMPETITION is sponsored by the NASA Florida Space Grant Consortium (FSGC) and the North East Florida Association of Rocketry (NEFAR). The University of South Florida's (USF) Society of Aeronautics and Rocketry (SOAR) will compete in two categories, with three teams. *Team Daedalus* will compete with a custom built hybrid one stage rocket in the "Closest to 2,000 feet" category. While the motor may be purchased for the actual competition, this paper will examine initial aspects of the rocket's aerodynamics by defining and developing the mathematical equations. The electronics and control system will be beyond the scope of this paper.

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Making Sense of the Motion

Identifying Forces and Moments

The following forces act on the rocket:

Thrust (T):

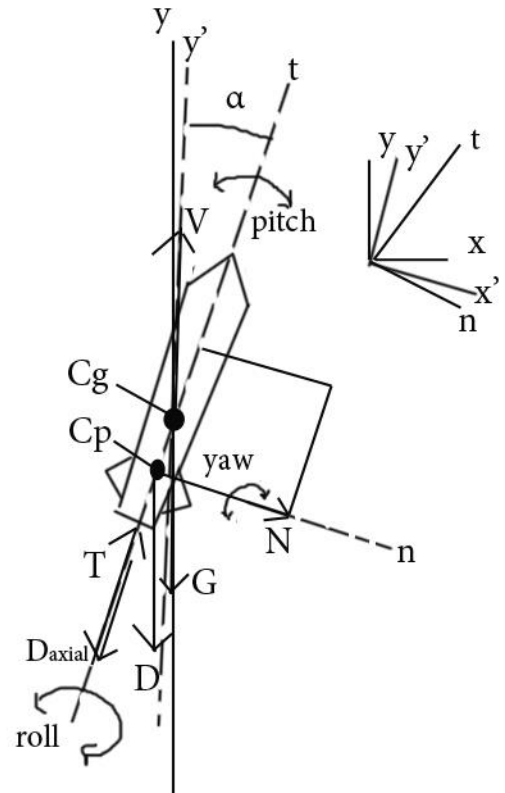
- Generated from the motor exhausting gasses in the opposite direction.
- Directly proportional to the exit velocity of the nozzle and the mass / time.
- Aligned to the center line, to avoid producing an angular moment.

Gravity (G):

- Sum of forces and moments of each component is the center of gravity, C_g .
- No angular moment is produced.

Aerodynamic (D, N):

- Produces net forces and angular moments.
- Axial Drag: component of drag parallel to velocity that opposes motion.
(in the picture, it would be $D \cos \alpha$)
- *Normal force*: provides stabilization by generating a corrective moment
 - *Pitch*: moment about the lateral axis
 - *Yaw*: moment about the vertical axis
 - *Roll*: moment about the longitudinal axis



Stability:

Calculating Center of Pressure and Coefficients

A rocket will fly straight into the airflow, but an imbalance of forces will cause the body to translate, and an imbalance of moments will cause the body to rotate. When there is thrust misalignment, a fin in incorrectly placed or a gust of wind, the rocket may tilt from its original, vertical orientation. If this happens, the rocket will fly at a new angle, changing the aerodynamics of its path. The angle of attack, α , is the angle between the centerline and the vertical component of its velocity.

A stable rocket will continuously try to correct its course and return to $\alpha = \text{zero}$. However, if the α increases too much, the C_p will move upwards and could meet or overtake the C_g , which will negate the corrective motions entirely, and make the rocket unstable.

Each component of the rocket will have its own normal forces acting perpendicular to the centerline; however, they can be summed up and expressed as acting through the center of pressure, C_p . If the C_p is located 1-2 calibers (1 caliber is the max body diameter) aft of the center of gravity, C_g , the rocket will act to correct the path by producing a moment. The stability margin is the distance between the C_p and C_g (measured in calibers).

The center of pressure, C_p , is the point on the body where the normal force is the only force that produces a pitching moment. It is the point where there is as much normal force ahead as behind it; a balancing point separate from the center of gravity. In order to develop an equation for C_p , several factors must be considered, including the several coefficients. Here is the plan of derivation:

1. Identify The Normal Force Coefficient
2. Identify The Pitching Moment Coefficient
3. Moving the Pitching Moment Coefficient
4. Set to 0 to Find the Center of Pressure location, X
5. Use l'Hopital to find Center of Pressure (Barrowman's Method)

The normal force for an axially symmetric body in subsonic flow:

$$N(x) = \rho v_0 \frac{\partial}{\partial x} [A(x)w(x)], \quad (1)$$

Where A_x is the cross-sectional area of the body

$w(x)$ is called the local downwash, and is given as function of α

$$w(x) = v_0 \sin \alpha$$

A normal force $N(x)$ at position x produces a pitching moment at the nose tip:

$$m_{pitch}(x) = xN(x) \quad (2)$$

1. The normal force coefficient C_n :

$$C_n(x) = \frac{N(x)}{.5 \rho V_0^2 A_{reference}} = \frac{2 \sin \alpha}{A_{reference}} \frac{dA(x)}{dx} \quad (3)$$

$$C_n = \frac{N}{.5 \rho V_0^2 A_{reference}} = \frac{2 \sin \alpha}{A_{reference}} \int_0^l \frac{dA(x)}{dx} dx = \frac{2 \sin \alpha}{A_{reference}} [A(l) - A(0)]$$

Where $A_{reference}$ will be the base of the nose cone

2. The pitch moment coefficient C_m :

$$C_m(x) = \frac{m_{pitch}(x)}{.5 \rho V_0^2 A_{reference} d} = \frac{xN(x)}{.5 \rho V_0^2 A_{reference} d} \quad (4)$$

$$C_m = \frac{2 \sin \alpha}{A_{reference} * d} \int_0^l \frac{dA(x)}{dx} dx = \frac{2 \sin \alpha}{A_{reference} * d} [lA(l) - \int_0^l A(x) dx]$$

Where d is the diameter at a specific point

3. How to move the pitch moment coefficient to another point:

$$C_{m \text{ new}} * d = C_m * d - C_n \Delta x \quad (5)$$

Where Δx is the distance from nosecone along centerline

4. Find the location of C_p by setting $C_{m \text{ new}}$ to 0, and solving for X:

$$\begin{aligned} C_{m \text{ new}} * d &= C_m * d - C_n \Delta x, \\ 0 &= C_m * d - C_n \Delta x, \\ X &= \frac{C_m * d}{C_n} \end{aligned} \quad (6)$$

Where X is the distance of C_p from nosecone TIP on centerline

This equation is valid for when the angle of attack, α , is greater than 0.

As the $\lim_{\alpha \rightarrow 0} \alpha$: $\lim_{\alpha \rightarrow 0} C_m$, $\lim_{\alpha \rightarrow 0} C_n$.

5. Use l'Hopital's Rule and Barrowman's Method to Simplify Finding C_p

$$X = \frac{\frac{\partial C_m}{\partial \alpha}}{\frac{\partial C_n}{\partial \alpha}} * d \bigg|_{\alpha=0} = \frac{C_{m\alpha}}{C_{n\alpha}} * d \quad (7)$$

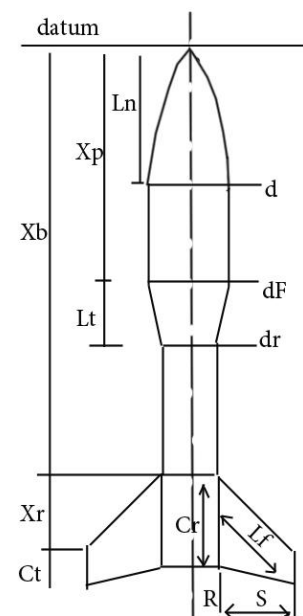
Barrowman's method is based on normal force coefficients and is only valid in the linear regime.

At small α , C_n and C_m can be approximated to be linear with α , therefore:

	For $\alpha > 0$	For $\alpha = 0$	
Normal force coefficient derivative =	$C_{n\alpha} = \frac{C_n}{\alpha}$	$C_{n\alpha} = \frac{\partial C_n}{\partial \alpha} \bigg _{\alpha=0}$	(8)

Pitch moment coefficient derivative =	$C_{m\alpha} = \frac{C_m}{\alpha}$	$C_{m\alpha} = \frac{\partial C_m}{\partial \alpha} \bigg _{\alpha=0}$	(9)
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In 1967, James S. Barrowman of the National Aeronautics and Space Administration's Sounding Rocket Branch submitted a document entitled 'The Practical Calculation of the Aerodynamic Characteristics of Slender Finned Vehicles'. It included new methods to calculate center of pressure. The Barrowman Method uses the coefficient derivatives to determine C_p . The first element of the Barrowman Method is that the normal force contribution of a straight, constant diameter body tube is zero. Only the nose, body diameter transition sections, and fins contribute to the normal force of the rocket. Calculations are performed with the normal force coefficients. All centers of pressure are referenced to datum zero, which is located at the nose.

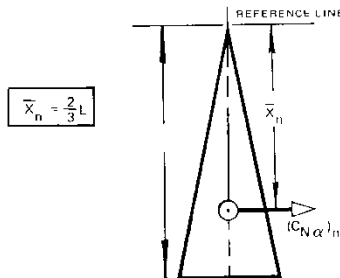


The centers of pressure of nose cones and body transition section are located by dividing the volume of the component by the area at the component's highest datum. This gives the center of pressure location from the component's base. A transform is required to calculate the datum of the CP.

L_n = length of nose	C_t = fin tip chord
d = diameter at base of nose	S = fin semispan
d_F = diameter at front of transition	L_f = length of fin mid-chord line
d_R = diameter at rear of transition	R = radius of body at aft end
L_t = length of transition	X_r = radius of body at aft end
X_p = dist from tip of nose to front of transition	X_b = dist between fin root leading edge and tip
Cr = fin root chord	N = number of fins

CONICAL NOSE

The distance from the tip of the nose to the center of pressure location of a cone-shaped nose is,



Body Terms

The normal force contribution of a straight, constant diameter body tube is zero.

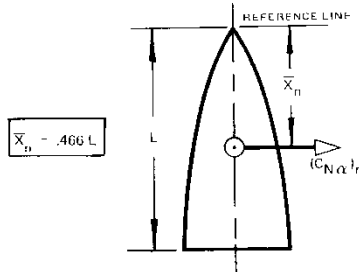
Conical Transition Terms

$$(C_n)_{Trans} = 2 \left[\left(\frac{d_R}{d} \right)^2 - \left(\frac{d_F}{d} \right)^2 \right]$$

$$X_{Trans} = X_p + \frac{L_t}{3} \left[1 + \left(\frac{1 - \frac{d_F}{d_R}}{1 - \left(\frac{d_F}{d_R} \right)^2} \right) \right]$$

OGIVE NOSE

The distance from the tip of the nose to the center of pressure location of ogive-shaped nose is,



Nose Cone Terms

$$(C_n)_{Nose} = 2 \quad (\text{for ALL nosecones}) \quad (10)$$

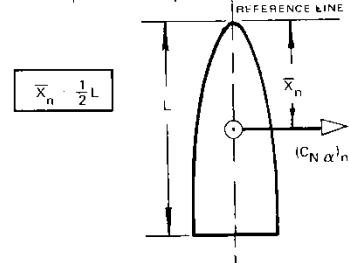
$$X_{Nose} \text{ (for Cone): } = 0.666 L_n \quad (11)$$

$$X_{Nose} \text{ (for Ogive): } = 0.466 L_n$$

$$X_{Nose} \text{ (for Ogive): } = 0.466 L$$

PARABOLIC NOSE

The distance from the tip of the nose to the center of pressure location of a parabolic nose is,



Fin Terms

$$(C_n)_{Fin} = \left[1 + \frac{R}{S+R} \right] \left[\left(\frac{4N \left(\frac{S}{d} \right)^2}{1 + \sqrt{1 + \left(\frac{2L_f}{C_r + C_t} \right)^2}} \right) \right] \quad (12)$$

$$X_{Fin} = X_b + \frac{X_r}{3} \frac{(C_r + 2C_t)}{(C_r + C_t)} + \frac{1}{6} \left[(C_r + C_t) - \frac{(C_r * C_t)}{(C_r + C_t)} \right] \quad (13)$$

Finding the Center of Pressure: Sum up coefficients: $(C_n)_{Total} = (C_n)_{Nose} + (C_n)_{Trans} + (C_n)_{Fin}$

Find CP Distance from Nose Tip:
$$\bar{X} = \frac{(C_n)_{Nose} * X_{Nose} + (C_n)_{Trans} * X_{Trans} + (C_n)_{Fin} * X_{Fin}}{(C_n)_{Total}} \quad (14)$$

The assumptions that must be met if applying Barrowman's Method:

1. The angle of attack is very close to zero. (Less than 10 degrees)
2. The speed must be much lower than the speed of sound (600 feet/s)
3. The air flow around the body is smooth and doesn't change rapidly
4. The rocket is thin compared to its length
5. The nose of the rocket comes smoothly to point
6. The rocket is an axially symmetric rigid body
7. The fins are thin flat plates

Stability:

Calculating Reynolds Number and Air Flow Considerations

Air flows smoothly around the streamlined body in layers, each of which has a different velocity. The layer closest to the surface stays with the surface and has zero velocity. Each additional layer gradually increases the speed until the free-stream velocity is reached. This flow is said to be laminar and to have a laminar boundary layer. The thickness of the boundary layer will increase with the distance the air has traveled along the surface. At some point a transition occurs and the layers of air begin to mix. The boundary layer becomes turbulent and thickens rapidly. A turbulent boundary layer induces a notably larger skin friction drag than a laminar boundary layer.

This is why it is necessary to consider how much of the flow around the rocket is laminar, and where it becomes turbulent. The point at which the flow becomes turbulent is the point that has a local critical Reynolds number (Re_c). The Reynolds number expresses the ratio of inertial (resistant to change or motion) forces to viscous forces, and is dimensionless. The Reynolds Number can be used to determine if flow is laminar, transient or turbulent.

$$RN = Re = \frac{\text{density} * \text{velocity} * \text{length}}{\text{viscosity}} \quad Re_{critical} = \frac{V_0 * x}{\nu} \quad (15)$$

Where V_0 is the free-stream air velocity,

x is the distance along the body from the nose cone tip

ν (nu) is the kinematic viscosity of air, $\approx 1.615 * 10^{-4} \text{ ft}^2/\text{s}$

$Re_{critical}$ is approximately 500,000

Most of the aerodynamic properties of rockets vary with the velocity of the rocket. An important parameter is the Mach number, which is the free stream velocity of the rocket divided by the local speed of sound. In subsonic flight all of the airflow around the rocket occurs below the speed of sound. This is the case for approximately $M < 0.8$. At very low Mach numbers air can be effectively treated as an incompressible fluid. The Team Daedalus rocket will be well under the speed of sound (600 feet/second).

$$\text{Mach} = M = \frac{V_{stream}}{c} \quad (16)$$

Where V_{stream} is the free-stream air velocity,
 c is the local speed of sound

According to the Glenn Research Center NASA website, If the air can be treated as an incompressible fluid, then to model flow, the density is constant and can be removed from Euler's continuity equation, though it is not ideal for future use for larger, more complex rocket modeling.

"We have two versions of the Euler Equations which describe how the velocity, pressure and density of a moving fluid are related. They are actually simplifications of the more general Navier-Stokes equations of fluid dynamics. The Euler equations neglect the effects of the viscosity of the fluid which are included in the Navier-Stokes equations. A solution of the Euler equations is therefore only an approximation to a real fluids problem. For some problems, like the lift of a thin airfoil at low angle of attack, a solution of the Euler equations provides a good model of reality. For other problems, like the growth of the boundary layer on a flat plate, the Euler equations do not properly model the problem."

<http://www.grc.nasa.gov/WWW/BGH/eulereqs.html>

$$2 \text{ Dimensional, Steady Form Continuity:} \quad \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (17)$$

$$\text{Simplified Incompressible Form Continuity:} \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (18)$$

Where u (x component) and v (y component)
are the components of velocity, and ρ is density.

Then Euler's Steady Form momentum equations can also be factored and simplified.

2 Dimensional, Steady Form

$$\text{X - Momentum: } \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = - \frac{\partial P}{\partial x}, \quad \text{Y - Momentum: } \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = - \frac{\partial P}{\partial y} \quad (19)$$

Simplified Incompressible Form Continuity

$$\text{X - Momentum: } u \frac{\partial(u)}{\partial x} + v \frac{\partial(u)}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x}, \quad \text{Y - Momentum: } u \frac{\partial(v)}{\partial x} + v \frac{\partial(v)}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial y} \quad (20)$$

Where P is pressure, u (x component) and v (y component) are the components of velocity, and ρ is density.

Stability:

Calculating Center of Gravity

According to Barrowman, the simplest way to calculate the center of gravity is the follow 5 steps:

1. Determine the weight of each individual component of the body design.
2. Find the center of gravity for each component.
 - a. Cylindrical objects (body tubes, engines, couplers, etc) have C_g at their midpoints.
 - b. Nose cones have C_g at 1/3 total length from the wide end.
 - c. The parachute, shock cord and lines have C_g at the middle of length when packed in the body tube.
3. Measure the distance between the nose tip and the center of gravity of each component.
4. Add the weights of the individual components to get the overall weight of the body

$$W_{Body} = W_1 + W_2 + W_3 + \dots$$

5. Use the formula:
$$\overline{(X_{CB})}_{Body} = \frac{W_1*(X_{Cg})_1 + W_2*(X_{Cg})_2 + W_3*(X_{Cg})_3}{W_{Body}} \quad (21)$$

Where $\overline{(X_{CB})}_{Body}$ is the body C_g location, measured from the nose tip.

And each (X_{Cg}) represents the distance from the nose tip and the C_g of the component.

Stability:

Calculating Drag

Drag resists the movement of the rocket relative to the air. At subsonic speeds, drag is produced by skin friction, pressure distribution around the components, or parasitic drag from launch lugs on the rocket. It increases proportionally with α , and is minimized when $\alpha=0$, therefore it is important to use the C_p to calculate the stability margin. Having a large enough margin will help keep the rocket self-correcting, reducing drag. However, if the margin is too large, on a windy day the rocket will consistently arc overhead instead of flying vertically. This is called weather-cocking. To avoid it, the standard is to ensure the stability margin is at least equal and preferably a little larger than the greatest diameter in the rocket. "One caliber stability" means that the C_p is one maximum body diameter behind the C_g .

Drag Equation:
$$D = \frac{1}{2} C_D \rho V_0^2 A_{reference} \quad (22)$$

Drag coefficient:
$$C_D = \frac{D}{.5 \rho V_0^2 A_{reference}} \quad \text{and} \quad C_{D0} = C_{A0} \quad (23)$$

Where ρ is the density of the fluid, in slugs/ft³
(not the radius of curvature)

The C_D is used to describe how the shape of the rocket and its angle influence drag. It is dimensionless, and anything that moves in air has a C_D . At $\alpha=0$, The total drag coefficient (C_D) and axial drag coefficient (C_A) coincide, but at any other angle, they are considered separately. When $C_{D0} = C_{A0}$ it is called Zero Lift Drag Coefficient, and it has several parts. Each rocket component will contribute some drag to the calculation.

- Base drag, C_{DB} , is only considered in the coasting phase, because at launch, the base pressure is equal to the atmospheric, so no pressure imbalance exists.

$$C_{DB \text{ booster}} = 0 \quad (24)$$

$$C_{DB \text{ coasting}} = \frac{0.029}{\sqrt{C_{DNosecone} + C_{DBody}}} \quad (25)$$

- Skin friction drag arises from contact of the body and fin with the airflow. The area in contact is the reference area, and it is called wetted area.

$$C_{skin \text{ friction}} = \frac{D_{friction}}{.5 \rho V_0^2 A_{wetted}} \quad (26)$$

It is a function of the Reynolds number and surface roughness. For a turbulent flow with a smooth surface with the surface roughness completely imbedded in a laminar layer:

$$Re_{critical} = 51 \left(\frac{R_s}{L}\right)^{-1.039} \quad (27)$$

Where R_s is the approximate height of the surface in micrometers

If the Reynolds number is below 100,000, $C_{skin \text{ friction}} = .0148$

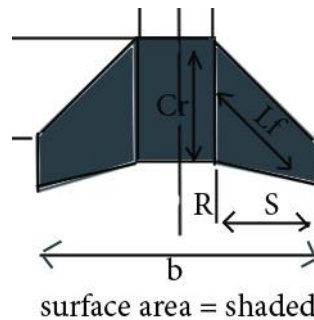
If it is above 100,000 but below $Re_{critical}$, $C_{skin \text{ friction}} = \frac{1}{1.50 \ln Re - 5.6}$

If it exceeds the critical value: $C_{skin \text{ friction}} = 0.032 \left(\frac{Re_s}{L}\right)^{0.2}$

- Fin drag requires several equations, and several others (included induced drag) have been omitted as being beyond the scope of this paper:

$$\lambda t = \frac{C_t}{C_r} = \frac{\text{tip chord}}{\text{root chord}} = \text{taper ratio} \quad (28)$$

$$\text{Aspect ratio} = AE = \frac{b \times b}{\text{Surface area of fins and connection}} \quad (29)$$



$$\text{Thickness ratio} = \frac{t}{c} = \frac{\text{thickness}}{\text{chord}} \quad (30)$$

$$C_{DOFins} = \frac{D_{fins}}{\frac{1}{2} \rho v^2 \text{planform area}} \quad (31)$$

$$C_{DOFins} = 2 * C_{\text{skin friction}} [1 + 2 \frac{t}{c}] \quad (32)$$

- Nose cone drag exists, but is much smaller than the skin friction drag.

“It is notable that even a slight rounding at the joint between the nose cone and body reduces the drag coefficient dramatically. Rounding the edges of an otherwise flat head reduces the drag coefficient from 0.8 to 0.2, while a spherical nose cone has a coefficient of only 0.01. The only cases where an appreciable pressure drag is present is when the joint between the nose cone and body is not smooth, which may cause slight flow separation.”

OpenRocket Technical Document, Sampo Niskanen

$$C_{D \text{ Nose}} + C_{D \text{ Body}} = 1.02 C_{\text{skin friction}} [1 + \frac{1.5}{(\frac{L}{d})^{3/2}}] \frac{\text{Wetted surface area}}{\text{Area of the body}} \quad (33)$$

Where L/d is the length to diameter ratio

- Parasitic Drag is what develops from having one or two launch lugs attached to the body of the rocket. It may be modeled as a solid cylinder, instead of a hollow one, next to the main rocket body.

$$C_{D \text{ Launch Lugmax}} = 1.2 \frac{\text{Surface area of lug}}{\text{Surface area of body tube}} \quad (34)$$

Where Surface area of lug = $\pi d_{in} l + \pi d_{out} l$

The total drag coefficient (C_D) is obtained by scaling all the drag coefficients to a common reference area and making a summation:

$$C_{D0} = \sum_T \frac{A_{Total}}{A_{reference}} (C_D)_{Total} \quad (35)$$

Where $\alpha = 0$

When $\alpha \neq 0$, $C_{D0} \neq C_{A0}$. More area interacts with the airflow, pressure gradients change and vortices at the fins develop. The axial drag coefficient (C_A) must be considered separately. All of these are valid for small α – usually less than 10° , but with an upper limit around 17° .

For $\alpha = 0^\circ$, $C_A = 1$

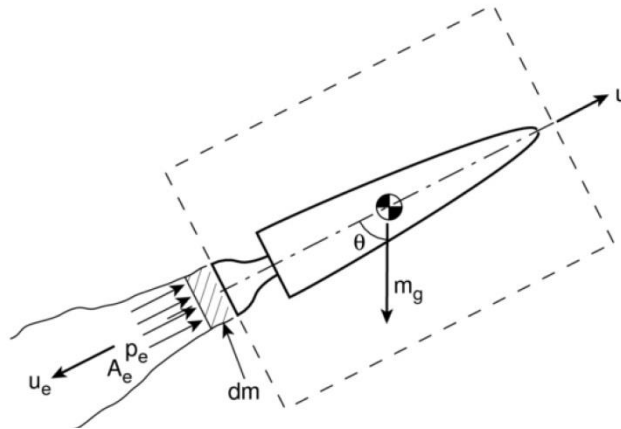
For $\alpha = 17^\circ$, $C_A = 1.3$

Stability:

Calculating Thrust

The calculation of thrust is important and will need to be completed before the final Daedalus design is finished, however at this time, the chemistry of the oxidizer and fuel grain and data on the final nozzle design are not available. Also, the vapor pressure of the nitrous oxide is highly dependent on temperature, which affects the thrust of the motor. This may cause some variation in the thrust between true flight and motor tests. Until the data is available for analysis, the model will be based on the maximum thrust allowable for the contest: a “G” motor, capable of producing 160 N of thrust.

However some of the equations that will be developed to determine thrust at each phase include:



$$F = ma = m \frac{dv}{dx} : \text{only for a closed system: need more general equation} \quad (36)$$

$$\underbrace{\frac{d}{dt}(m_b v + \int \rho(u + v)dV)}_{\text{Rate of change in rocket momentum}} = \underbrace{(P_{out} - P_{atm})A_e + F_{Drag} - F_{gravity}}_{\text{External Forces}} + \underbrace{\dot{m}(u_e + v)}_{\text{Momentum flow through outlet}} \quad (37)$$

Where $(u+v)$ is velocity vector components relative to the ground

The mass flow through the inlet:

$$\dot{m} = \frac{d \text{ mass total}}{dt} = \rho_{exit} u_{exit} A_{exit} \quad (38)$$

$$\frac{d}{dt}(m_b v + \int \rho(u + v)dV) = \text{mass}_{total} a + \dot{m} v + \frac{d}{dt}(\int \rho(u)dV), \quad (39)$$

Substituting:

$$F_{int} = -\frac{d}{dt}(\int \rho(u)dV) \quad (40)$$

$$F_{thrust} = (P_{out} - P_{atm})A_e + \dot{m} u_e \quad (41)$$

$$acceleration = a = \frac{F_{thrust} + F_{drag} + F_{int}}{mass_{total}} - g \quad (42)$$

$$\frac{dv}{dt} = a; \frac{dy}{dt} = v, \quad (43)$$

Also, because the propellant makes up such a large proportion of the total mass, the changing mass must be addressed.

$$\text{Empty Mass} = m_e = \text{payload mass} + \text{structural mass} \quad (44)$$

$$\text{Full Mass} = m_f = \text{payload mass} + \text{structural mass} + \text{propellant mass} \quad (45)$$

$$m_f = m_e + \text{propellant mass}$$

$$\text{Structural coefficient} = \varepsilon = \text{structural mass} / (\text{propellant mass} + \text{structural mass}) \quad (46)$$

$$\text{Payload ratio} = \lambda = \text{payload mass} / (\text{full mass} - \text{payload mass}) \quad (47)$$

$$\text{Propellant mass ratio} = MR = \text{full mass} / \text{empty mass} \quad (48)$$

$$MR = 1 + \text{propellant mass} / \text{empty mass}$$

$$MR = \frac{1+\lambda}{1+\varepsilon}$$

Differential Position Equations

Position

Orientation

$$\ddot{y}(t) = \dot{v}(t) = a(t) \quad \ddot{\phi}(t) = \dot{\omega}(t) = \alpha(t) \quad (49)$$

$$\dot{y}(t) = v(t) \quad \dot{\phi}(t) = \omega(t) \quad (50)$$

Presented as a first order, nonlinear ordinary differential equation, where y is a vector containing the position and orientation of the rocket:

$$y' = f(y, t)$$

Using the Runge-Kutta 4 method

Definition: A method of integrating ODEs that involves using steps at the middle of an interval to cancel out lower order error terms.

Formula: $k_1 = h f(x_n, y_n)$

$$k_2 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$$

$$k_3 = h f(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$$

$$k_4 = h f(x_n + h, y_n + k_3)$$

$$y_{n+1} = y_n + \frac{1}{6}k_1 + \frac{1}{3}k_2 + \frac{1}{3}k_3 + \frac{1}{6}k_4 + O(h^5)$$

Applied: $k_1 = f(y_0, t_0)$

$$k_2 = f(y_0 + \frac{1}{2}k_1\Delta t, t_0 + \frac{1}{2}\Delta t)$$

$$k_3 = f(y_0 + \frac{1}{2}k_2\Delta t, t_0 + \frac{1}{2}\Delta t)$$

$$k_4 = f(y_0 + k_3\Delta t, t_0 + \Delta t)$$

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\Delta t$$

$$\varphi_1 = \varphi_0 + \omega\Delta t$$



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HYBRID MOTOR ROCKET COMPETITION

2014-2015 HYBRID MOTOR HIGH POWERED ROCKET COMPETITION

Sponsored by the NASA Florida Space Grant Consortium (FSGC) and the North East Florida Association of Rocketry (NEFAR)

The objective of the competition is to build and launch a hybrid powered rocket. There are two categories of competition to choose from. The first category consists of launching a hybrid rocket to the maximum altitude. The second category challenges the teams to fly their rocket closest to 2000 feet in altitude. There must be at least two teams competing in each category. If there is only one team, that team will be asked to move to the other category.

The rocket can be built from scratch or from a kit.

The engine must be a hybrid motor rated "G" or from a lower class. The engine can be built from scratch or purchased from a company. NOTE: If the motor is built from scratch or is modified in anyway, a minimum of 2 documented motor tests must be done to demonstrate the safety, quality, and performance of the motor. Documentation must be performed and submitted two weeks before launch and must show thrust curves, impulse, burn-time, etc. from the 2 tests (please confer with the judges prior to making any modifications to the motor).

Points will be awarded for the phases of the competition. The successful flight is worth 80% of the total points and the teams Engineering Notebook report is worth 20% of the total points. The points for each part are as follows:

1. Points for Flight
 1. 100 pts for highest or closest to altitude
 2. 90 pts for 2nd highest or closest to altitude
 3. 80 pts for 3rd highest or closest to altitude
 4. 70 pts for 4th highest or closest to altitude
 5. 0-10 pts for self built motor
 6. 0-5 pts for self built rocket
2. Points for Engineering Notebook
 1. Rocksim or other software Simulations (30 pts)
 2. Engineering Data (70 pts)

The engineering notebook will be a bound notebook (Composition type noteBook) which will have all of the team's engineering data, calculations, drawings and sketches, test results, notes, ideas, meeting notes, etc. This notebook will be returned to the teams on flight day. **NOTE: The notebook is NOT a formal final report. We are looking for your project/laboratory workbook.**

This competition is open to any university or community college team in Florida, both public and private.

Part I – Proposal

The faculty advisor of the university team must submit a 2 page proposal with a budget of up to \$1,000.

If a team is planning to enter both categories, please submit separate proposals (maximum 2 pages each)

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The proposal must include the following:

1. Team Name
2. Project Manager Name and Email Address
3. Name and Email Address of the Project Manager's Alternate
4. All team members names and their email addresses in addition to the following:
 1. Status (e.g. sophomore, junior, senior etc.)
 2. Hometown
 3. Gender
 4. Race (needed for reporting to funding authority)
5. Category or categories the team plans to compete in (maximum altitude and/or closest to 2000 feet in altitude).
6. Detailed Budget.

All teams that submit a proposal will be able to take part in the competition and compete for the prizes. At least 6 teams (from both categories) will be selected and awarded up to \$1,000 to build the rocket (detailed budget must be provided in the proposal). **For teams that design their own engines, static testing and data from two test launches is expected.** The funds will be provided as a cost reimbursable grant to the faculty advisor. The funds can be used for supplies, motors, kits and travel. Salary and capital expenditure is not allowed. Indirect costs are not allowed.

Part II – Reports & Flight

Teams will then build and test their rockets for flight and submit their engineering notebook due approximately 2 weeks before the launch. Also, the program manager or their alternate representative from each team will be required to submit every 2 weeks a "progress and accomplishment report" of no more than 3/4 page text to the PBWorks website @ <http://hybridrocket.pbworks.com/>. Reports should include work done plus attachments including parts lists, photos, etc. NOTE: There are no points awarded for submitting the Progress Reports however, points will be deducted for non-submittal and late reports.

Also, each team will be required to submit to the PBWorks website @ <http://hybridrocket.pbworks.com/> a Hazard Analysis and a Failure Modes & Effects Analysis by the time listed in the **Timeline below**. The Hazard Analysis should focus on the handling and use of the nitrous oxide and any pyrotechnic systems or materials. The Failure Modes & Effects Analysis should focus on what kinds of things could go wrong with your launch equipment and rocket, as well as, what you have done to mitigate or reduce the identified failure modes. These reports should be no more than 4 text pages in length, tables and graphs are not included in page count. They should be updated and resubmitted as your designs evolve. The reports are to show that you are ready to test and fly your rockets and motors safely. Failure to submit these reports may result in your being removed from the competition.

LAUNCH DAY

Teams will have their rockets and motors inspected for safety by a NEFAR representative just before launch. NEFAR will sponsor the launch at the club site in Bunnell. Results of launch must be into the judge by 3:00 pm on the day of the launch; judge will leave site at 3:15 pm. **NOTE: We are not responsible for problems at the NEFAR launch site. Be there early. Don't wait until it is too late to launch.** To be awarded points for flight you must have a successful flight; i.e. Launch, deployment of recovery system, and controlled landing. All other flights will be judged on a case by case base. **NOTE: rockets deemed unsafe will not be allowed to fly in the competition until fixed and approved.**

NEFAR Website – <http://www.nefar.net/>

Part III – Additional funding

The winning teams from each category will receive additional funding according to the following chart. These additional funds will be provided to the winning team(s) faculty advisor to defray their expenses incurred for travel to the competition location.

Place	Maximum Altitude Category	Closest to 2000 ft. Category
1 st Place	\$500.00	\$750.00
2 nd Place	\$300.00	\$450.00
3 rd Place	\$100.00	\$200.00

Time line

Sept. 12, 2014: Proposal with Budget, Deadline

Sept. 26, 2014: Announcement of winners and grant awarded to faculty advisor

Oct. 10, 2014: First of the every two week report due.

Nov. 14, 2014: Hazard Analysis, Failure Modes & Effects Analysis due.

Apr. 3, 2015: Engineering Notebook due at Florida Space Grant Offices.

Apr. 11, 2015: Launch (May 9th: alternate date)

Altitude Determination

Altimeters

A recording barometric altimeter must be used to record data for competition. The launch site should be considered zero altitude and the altimeter should be calibrated to zero, it is up to flier to provide proof of a properly calibrated altimeter to the Judge upon request.

Altimeters with altitude sensors other than barometric sensors, such as accelerometers or magnetic apogee detection, may be used to deploy the recovery systems. However, they are prohibited from use in determining the actual altitude.

Determining Actual Altitude

The actual flight profile will be determined by the competition judges. The graph or other flight profile display provided by a recording device will be examined for accuracy. If it is shown that a sudden peak in altitude is attributable to the ejection charge, that peak will be not be used to determine the recorded altitude. The altitude just prior to or just after that sudden peak will be the official recorded altitude.

Launch Rails & Firing Electronics Requirements

Teams should provide their own launch rails/pads and firing electronics and if requested must be inspected for safety by a NEFAR representative. **NOTE THIS REQUIREMENT: firing electronics must be at least 500 feet away from launch rails/pads. Firing electronics should incorporate at least one safety switch to prevent accidental ignition of rocket during setup. Please insure that you have enough current available to ignite the motor with 500 foot of cable.** If you wish to use NEFAR launch equipment please contact Robert Eppig for our NEFAR representative contact information – to check if what you need is available. Please check early if you wish to use NEFAR equipment-THERE IS NO GUARANTEE THAT WHAT YOU NEED IS AVAILABLE.

Static Judging

For the teams that build their rocket and/or engine from scratch, their scores from the judges will reflect the originality and performance of the rocket and/or the engine

Motor Class Total Impulse

G or less: 160 Newton-seconds or less

Submit Proposal and Engineering Notebook to:

Gene Tavares, 407-823-6173, Eugene.Tavares@ucf.edu

Submit technical questions to:

2013-14 PARTICIPATING UNIVERSITIES

1. University of Central Florida (1 team)
 2. University of Florida (1)
 3. University of Miami (1)
 4. University of West Florida (1)
 5. Daytona State College (1)
 6. Florida International University (1)
 7. Florida Institute of Technology (2)
 8. Embry-Riddle Aeronautical University (2)
 9. University of South Florida (3)
-

2013-14 WINNERS

Maximum Altitude

First Place: Florida Institute of Technology

Second Place: University of West Florida

Third Place: University of Florida

Closest to 2000 feet

First Place: University of Central Florida

Second Place: Florida Institute of Technology

Third Place: Florida International University

2012-13 Participating Universities

1. University of Central Florida
2. University of Florida
3. University of Miami
4. Embry-Riddle Aeronautical University
5. Florida Institute of Technology

2012-13 Winners

Maximum Altitude

First Place: University of Central Florida

Second Place: Florida Institute of Technology

Third Place: University of Florida

Closest to 2000 feet

First Place: Florida Institute of Technology

Second Place: University of Florida

Third Place: University of Central Florida

2011-12 Winners

Maximum Altitude

First Place: University of Florida

Second Place: Florida Institute of Technology

Third Place: Embry-Riddle Aeronautical University

Closest to 2000 feet

First Place: Florida Institute of Technology

Second Place: University of Florida

Third Place: University of Central Florida

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Appendix B: Abbreviations

α = angle of attack

c = local speed of sound

C_D = drag coefficient

C_{D0} = zero lift drag coefficient, where axial component is $C_{A0} = C_{D0}$

C_{DOFins} = zero lift drag coefficient for the fins

C_{DBase} = coefficient of drag for the base

C_{DBody} = coefficient of drag for the body

$C_{DB booster}$ = coefficient of drag for the body during booster phase

$C_{DB coasting}$ = coefficient of drag for the body during coasting phase

$C_{D Launch Lugmax}$ = maximum coefficient of drag caused by launch lugs

$C_{DNosecone}$ = coefficient of drag for the nosecone

$C_{skin friction}$ = coefficient of skin drag friction

C_n = normal force coefficient

$(C_n)_{Nose}$ = normal force coefficient for the nose

$(C_n)_{Trans}$ = normal force coefficient for the transition areas

$(C_n)_{Fins}$ = normal force coefficient for the fins

C_m = pitch moment coefficient

C_r = fin root chord

C_t = fin tip chord

d = diameter at base of nose

d_F = diameter at front of transition

d_R = diameter at rear of transition

D = drag

λt = taper ratio

L_f = length of fin mid-chord line

L_n = length of nose

L_t = length of transition

M = Mach

m_{pitch} = pitch moment

n = number of fins

N = normal force

R = radius of body at aft end

ρ = density

$RN = RE$ = Reynolds Number

S = fin semispan

u = x component of velocity

v = y component of velocity

V_{stream} = free stream velocity

X = location of center of pressure on the centerline of the body

X_{Nose} = location of the center of pressure on the nose

X_{Trans} = location of the center of pressure on the transition areas

X_{Fins} = location of the center of pressure on the fins

X_b = dist between fin root leading edge and tip

$(\overline{X_{CB}})_{Body}$ = the body C_g location, measured from the nose tip.

X_p = dist from tip of nose to front of transition

X_r = radius of body at aft end

Appendix C: Glossary of Terms

- **aft** is the rear-ward end of something on a rocket. Ship terminology is often used because "top" and "bottom" are confusing as orientation changes. See also *forward*.
- **airframe** The rocket structure. This usually refers to just the cylindrical body tube, but may also refer to the entire body of the rocket.
- **altimeter** is a device which measures at least the maximum height a rocket reaches. These are often combined with circuitry to separate the rocket at apogee for recovery.
- **angle of attack (α)** the angle between the centerline and the vertical component of velocity.
- **apogee** is the highest point of a rocket flight. An ideal rocket flight opens the rocket and ejects the recovery system at apogee.
- **axial drag** component of drag parallel to velocity that opposes motion.
- **caliber** is the diameter of the main body tube of the rocket in question. For example, rockets are commonly 15-25 calibers in length. (This term comes from gunnery where caliber is the outside diameter of the shell.)
- **center of gravity (C.G.)** is the balance point of the rocket with the intended motor loaded.
- **center of pressure (C.P.)** is the balance point of aerodynamic forces on the rocket.
- **certification** The U.S. national rocketry organizations implement a system of certification, up to three levels.
- **corrective Moments**
 - **Pitch:** moment about the lateral axis
 - **Yaw:** moment about the vertical axis
 - **Roll:** moment about the longitudinal axis
- **ejection** The charge (or sometimes mechanical system) which opens the rocket at *apogee* to deploy the *recovery* system. The ejection delay is the amount of time between motor burnout and the deployment and it timed to occur at apogee.
- **engine** See *motor*.
- **forward** is the front end of something on a rocket.
- **impulse** is the measure of *thrust* over time (in Newton-seconds or pound-seconds of force). The "total impulse" of a motor is the amount of energy it provides to lift the rocket and the source of the letter designation ("A, B, C" and so on).
- **level N** "Level 1" refers to rockets which use H & I motors, "level 2" to J through L motors and "level 3" to M through O motors.
- **motor** The motive force making a rocket go. Solid fuel rockets use motors because there are no mechanical moving parts (they're not engines).
- **nose** The forward end of a rocket. The tapering part of the rocket is often referred to as a "nose cone," even though the shape is rarely conical.
- **Ogive** the roundly tapered end of a two-dimensional or three-dimensional object.
- **stability margin** the distance between C_p and C_{g_r} , measured in calibers
- **streamer** A recovery system for small rockets. Streamers are flat plastic, paper or cloth bands which are attached to the rocket and flap as the rocket comes down, slowing the descent.
- **thrust** is a measure of instantaneous force. The "average thrust" of a motor is the average amount it pushes on the rocket during its entire burn phase. Note that the motor generally produces different amounts of thrust as it burns and a graph of this is called a "thrust curve."

Appendix D: Listed Equations

$$N(x) = \rho v_0 \frac{\partial}{\partial x} [A(x)w(x)], \quad (1)$$

$$m_{pitch}(x) = xN(x) \quad (2)$$

$$C_n(x) = \frac{N(x)}{.5 \rho V_0^2 A_{reference}} = \frac{2 \sin \alpha}{A_{reference}} \frac{dA(x)}{dx} \quad (3)$$

$$C_n = \frac{N}{.5 \rho V_0^2 A_{reference}} = \frac{2 \sin \alpha}{A_{reference}} \int_0^l \frac{dA(x)}{dx} = \frac{2 \sin \alpha}{A_{reference}} [A(l) - A(0)]$$

$$C_m(x) = \frac{m_{pitch}(x)}{.5 \rho V_0^2 A_{reference} d} = \frac{xN(x)}{.5 \rho V_0^2 A_{reference} d} \quad (4)$$

$$C_m = \frac{2 \sin \alpha}{A_{reference} * d} \int_0^l \frac{dA(x)}{dx} dx = \frac{2 \sin \alpha}{A_{reference} * d} [lA(l) - \int_0^l A(x) dx],$$

$$C_{m\ new} * d = C_m * d - C_n \Delta x \quad (5)$$

$$X = \frac{C_m * d}{C_n} \quad (6)$$

$$X = \frac{\frac{\partial C_m}{\partial \alpha}}{\frac{\partial C_n}{\partial \alpha}} * d \big|_{\alpha=0} = \frac{C_{m\alpha}}{C_{n\alpha}} * d \quad (7)$$

$$C_{n\alpha} = \frac{C_n}{\alpha} \quad C_{n\alpha} = \frac{\partial C_n}{\partial \alpha} \big|_{\alpha=0} \quad (8)$$

$$C_{m\alpha} = \frac{C_m}{\alpha} \quad C_{m\alpha} = \frac{\partial C_m}{\partial \alpha} \big|_{\alpha=0} \quad (9)$$

$$(C_n)_{Nose} = 2 \text{ (for ALL nosecones)} \quad (10)$$

$$X_{Nose} \text{ (for Cone): } = 0.666Ln \quad (11)$$

$$X_{Nose} \text{ (for Ogive): } = 0.466Ln$$

$$X_{Nose} \text{ (for Parabolic): } = 0.5L$$

$$(C_n)_{Trans} = 2 \left[\left(\frac{dR}{d} \right)^2 - \left(\frac{dF}{d} \right)^2 \right] \quad (C_n)_{Fin} = \left[1 + \frac{R}{S+R} \right] \left[\left(\frac{4N \left(\frac{S}{d} \right)^2}{1 + \sqrt{1 + \left(\frac{2L_f}{C_r + C_t} \right)^2}} \right) \right] \quad (12)$$

$$X_{Trans} = X_p + \frac{L_t}{3} \left[1 + \left(\frac{1 - \frac{dF}{dR}}{1 - \left(\frac{dF}{dR} \right)^2} \right) \right] \quad X_{Fin} = X_b + \frac{X_r}{3} \frac{(C_r + 2C_t)}{(C_r + C_t)} + \frac{1}{6} [(C_r + C_t) - \frac{(C_r * C_t)}{(C_r + C_t)}] \quad (13)$$

Welcome to Rocketry

$$\bar{X} = \frac{(C_n)_{Nose} * X_{Nose} + (C_n)_{Trans} * X_{Trans} + (C_n)_{Fin} * X_{Fin}}{(C_n)_{Total}} \quad (14)$$

$$RN = Re = \frac{density * velocity * length}{viscosity}, \quad Re_{critical} = \frac{V_0 * x}{v} \quad (15)$$

$$Mach = M = \frac{V_{stream}}{c} \quad (16)$$

$$\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (17)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (18)$$

$$X - \text{Momentum: } \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} = - \frac{\partial P}{\partial x}, \quad Y - \text{Momentum: } \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} = - \frac{\partial P}{\partial y} \quad (19)$$

$$X - \text{Momentum: } u \frac{\partial(u)}{\partial x} + v \frac{\partial(u)}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x}, \quad Y - \text{Momentum: } u \frac{\partial(v)}{\partial x} + v \frac{\partial(v)}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial y} \quad (20)$$

$$\overline{(X_{CB})_{Body}} = \frac{W_1 * (X_{Cg})_1 + W_2 * (X_{Cg})_2 + W_3 * (X_{Cg})_3}{W_{Body}} \quad (21)$$

$$D = \frac{1}{2} C_D \rho V_0^2 A_{reference} \quad (22)$$

$$C_D = \frac{D}{.5 \rho V_0^2 A_{reference}} \quad \text{and when} \quad C_{D0} = C_{A0} \quad (23)$$

$$C_{DB \text{ booster}} = 0 \quad (24)$$

$$C_{DB \text{ coasting}} = \frac{0.029}{\sqrt{C_{DNosecone} + C_{DBody}}} \quad (25)$$

$$C_{skin \text{ friction}} = \frac{D_{friction}}{.5 \rho V_0^2 A_{wetted}} \quad (26)$$

$$Re_{critical} = 51 \left(\frac{R_s}{L}\right)^{-1.039} \quad (27)$$

$$\lambda t = \frac{c_t}{c_r} = \frac{tip \text{ chord}}{root \text{ chord}} = \text{taper ratio} \quad (28)$$

$$Aspect \text{ ratio} = AE = \frac{bxb}{Surface \text{ area of fins and connection}} \quad (29)$$

$$Thickness \text{ ratio} = \frac{t}{c} = \frac{thickness}{chord} \quad (30)$$

$$C_{DOFins} = \frac{D_{fins}}{\frac{1}{2} \rho v^2 planform \text{ area}} \quad (31)$$

$$C_{DOFins} = 2 * C_{skin \text{ friction}} \left[1 + 2 \frac{t}{c}\right] \quad (32)$$

Welcome to Rocketry

$$C_{D\text{ Nose}} + C_{D\text{ Body}} = 1.02 C_{\text{skin friction}} \left[1 + \frac{1.5}{\left(\frac{L}{d}\right)^{3/2}} \right] \frac{\text{Wetted surface area}}{\text{Area of the body}} \quad (33)$$

$$C_{D\text{ Launch Lugmax}} = 1.2 \frac{\text{Surface area of lug}}{\text{Surface area of body tube}} \quad (34)$$

$$C_{D0} = \sum T \frac{A_{\text{Total}}}{A_{\text{reference}}} (C_D)_{\text{Total}} \quad (35)$$

$$F = ma = m \frac{dv}{dx} : \text{only for a closed system: need more general equation} \quad (36)$$

$$\frac{d}{dt}(m_b v + \int \rho(u + v) dV) = (P_{\text{out}} - P_{\text{atm}})A_e + F_{\text{Drag}} - F_{\text{gravity}} + \dot{m}(u_e + v) \quad (37)$$

$$\text{mass flow through the inlet: } \dot{m} = \frac{d \text{ mass total}}{dt} = \rho_{\text{exit}} u_{\text{exit}} A_{\text{exit}} \quad (38)$$

$$\frac{d}{dt}(m_b v + \int \rho(u + v) dV) = \text{mass}_{\text{total}} a + \dot{m} v + \frac{d}{dt}(\int \rho(u) dV), \quad (39)$$

$$F_{\text{int}} = -\frac{d}{dt}(\int \rho(u) dV) \quad (40)$$

$$F_{\text{thrust}} = (P_{\text{out}} - P_{\text{atm}})A_e + \dot{m} u_e \quad (41)$$

$$\text{acceleration} = a = \frac{F_{\text{thrust}} + F_{\text{drag}} + F_{\text{int}}}{\text{mass}_{\text{total}}} - g \quad (42)$$

$$\frac{dv}{dt} = a; \frac{dy}{dt} = v, \quad (43)$$

$$\text{Empty Mass} = m_e = \text{payload mass} + \text{structural mass} \quad (44)$$

$$\begin{aligned} \text{Full Mass} &= m_f = \text{payload mass} + \text{structural mass} + \text{propellant mass} \\ m_f &= m_e + \text{propellant mass} \end{aligned} \quad (45)$$

$$\text{Structural coefficient} = \varepsilon = \frac{\text{structural mass}}{(\text{propellant mass} + \text{structural mass})} \quad (46)$$

$$\text{Payload ratio} = \lambda = \frac{\text{payload mass}}{(\text{full mass} - \text{payload mass})} \quad (47)$$

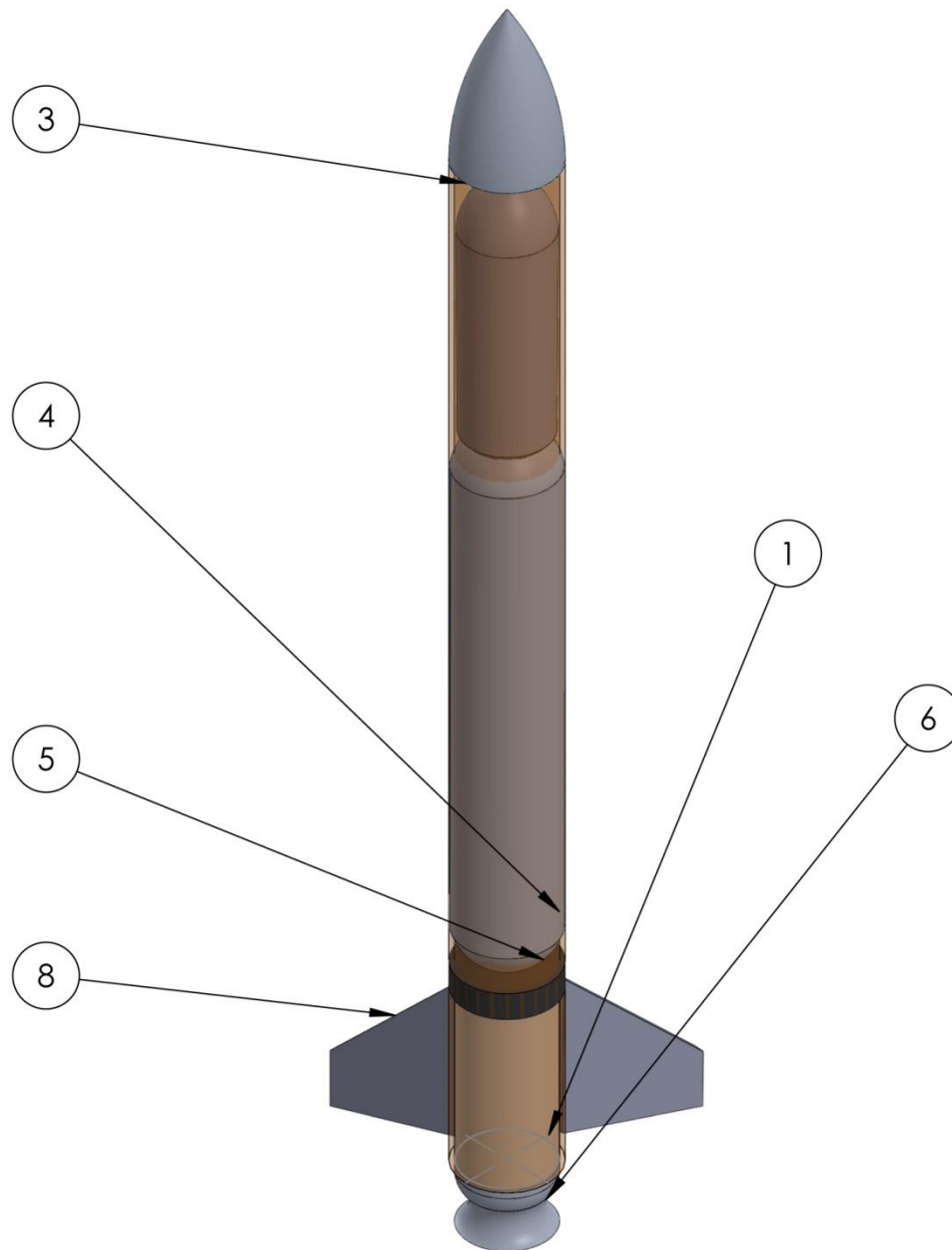
$$\text{Propellant mass ratio} = \text{MR} = \frac{\text{full mass}}{\text{empty mass}} \quad (48)$$

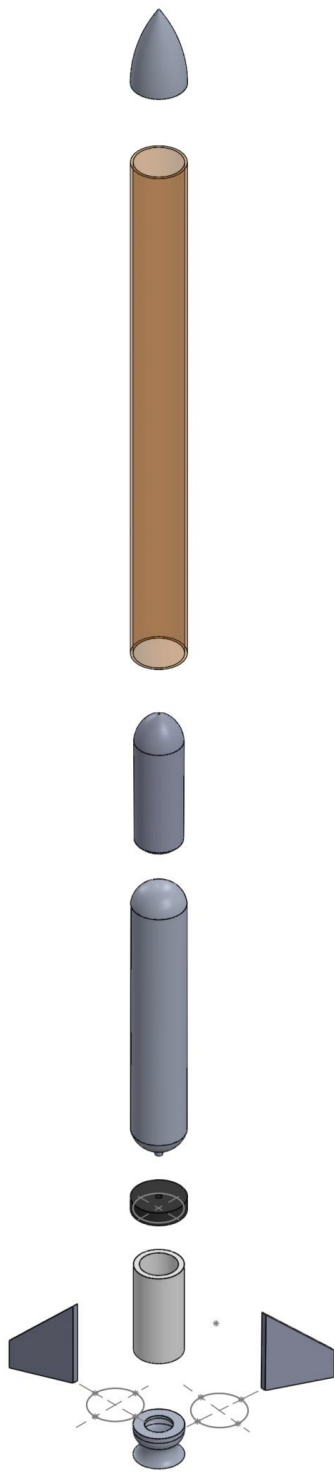
$$\text{MR} = 1 + \frac{\text{propellant mass}}{\text{empty mass}}$$

$$\text{MR} = \frac{1+\lambda}{1+\varepsilon}$$

$$\ddot{y}(t) = \dot{v}(t) = a(t) \quad \ddot{\phi}(t) = \dot{\omega}(t) = \alpha(t) \quad (49)$$

$$\dot{y}(t) = v(t) \quad \dot{\phi}(t) = \omega(t) \quad (50)$$





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